

# Proton Stability and Superstring $Z'$

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## Abstract

Recently it was argued that proton lifetime limits impose that the scale of quantum gravity must be above  $10^{16}$  GeV. By studying the proton stability in the context of realistic heterotic-string models, I propose that proton longevity necessitates the existence of an additional  $U(1)_{Z'}$  symmetry, which is of non-GUT origin and remains unbroken down to intermediate, or low, energies. It is shown that the realistic string models frequently give rise to  $U(1)$  symmetries, which suppress the proton decay mediating operators, with, or without, the possibility of R-parity violation. By studying the F- and D-flat directions, I examine whether the required symmetries remain unbroken in the supersymmetric vacuum and show that in some examples they can, whereas in others they cannot. The proton decay rate is proportional to the  $U(1)_{Z'}$  symmetry breaking scale. Imposing the proton lifetime limits I estimate the  $U(1)_{Z'}$  breaking scale and show that if substantial R-parity violation is present the associated  $Z'$  is within reach of forthcoming collider experiments.

# 1 Introduction

The proton longevity is one of the most important guides in attempts to understand the fundamental origin of the observed gauge and matter particle spectrum. While the Standard Model does not allow for the existence of renormalizable operators which can mediate proton decay, this is not the case in most of its theoretical extensions. Moreover, even if we assume that the Standard Model remains unmodified up to the cutoff scale set by quantum gravity, baryon and lepton number violating operators will in general be induced at that scale. In fact, recently it was argued, on general grounds, that proton lifetime limits impose that the cutoff scale must be above  $10^{16}\text{GeV}$  [1].

The proton longevity problem becomes especially acute in supersymmetric extensions of the Standard Model [2], which allow dimension four and five baryon and lepton number violating operators [3]. In the Minimal Supersymmetric Standard Model one imposes the existence of a global symmetry,  $R$ -parity, which forbids the dangerous dimension four operators, while the difficulty with the dimension five operators can only be circumvented if one further assumes that the relevant Yukawa couplings are sufficiently suppressed. However, in general, global symmetries are not preserved in quantum gravity [4, 5]. To satisfy proton lifetime constraints one must therefore assume the existence of a local discrete symmetry [6] or an explicit gauge symmetry. An example of such a symmetry is the gauged  $B - L$  symmetry which forbids the dimension four proton decay mediating operators of the MSSM.

Realistic superstring models provide a concrete framework to study in detail the issue of proton stability in the context of quantum gravity. Indeed the issue has been examined in the past by a number of authors [7, 8, 9]. The avenues explored range from the existence of matter parity at special points in the moduli space of specific models, to the emergence of non-Abelian custodial symmetries in specific compactifications.

The most realistic string models constructed to date are the models constructed in the free fermionic formulation [10]. This has given rise to a large set of semi-realistic models [11, 12, 13, 14, 15, 16, 17, 18], which differ in their detailed phenomenological characteristics, and share an underlying  $Z_2 \times Z_2$  orbifold structure [19]. The important achievements include: the natural emergence of three generations, which is correlated with the structure of the underlying  $Z_2 \times Z_2$  orbifold; The  $SO(10)$  embedding of the Standard Model spectrum, yielding the canonical  $SO(10)$  normalization for the weak hypercharge. Recently, it was further demonstrated that free fermionic construction also gives rise to models in which the low energy states, which carry Standard Model charges, consist solely of the spectrum of the Minimal Supersymmetric Standard Model [17]. The realistic free fermionic models therefore provide a concrete and viable framework to study the proton lifetime problem. In this context past investigations have examined several possibilities that may explain the proton longevity. For example, ref. [20] stipulated the possibility that the  $U(1)_{Z'}$  which is

embedded in  $SO(10)$  remains unbroken down to the TeV scale, and consequently the problematic dimension 4 operators are adequately suppressed. In ref. [7] the existence of superstring symmetries which naturally suppress the proton decay mediating operators was studied, while in ref. [16] it was shown that the free fermionic string models occasionally give rise to non-Abelian custodial symmetries, which forbid proton decay mediating operators to all orders of non-renormalizable terms. These proposals, however, fall short of providing a satisfactory solution. The reason being that these proposals are, in general, exclusive to the generation of light neutrino masses through a see-saw mechanism. For example, the absence of the  $SO(10)$  126 representation in string models necessitates that the  $SO(10)$   $U(1)_{Z'}$  be broken at a high scale, rather than at a low scale. Similarly, to date, the existence of the custodial non-Abelian symmetries seems to be exclusive to the generation of a see-saw mass matrix. I also remark that the presence of additional gauge bosons in non-realistic string models as been noted in ref. [21], as well as a suggestion that the low energy data hints on the existence of an additional  $Z'$  with stringy characteristics [22].

The above discussion highlights both the importance and difficulty of finding a robust and satisfactory solution to the proton stability problem. The solution which is advocated in this paper is that unification of gravity and the gauge interactions necessitates the existence of an additional  $U(1)$  symmetry, beyond the Standard Model, which remain unbroken down to low or intermediate energy. Furthermore, the required  $U(1)$  symmetry is not of the type that arises in  $SO(10)$  or  $E_6$  GUTs. Invariance under the extra  $U(1)$  forbids the proton decay mediating operators, which can be generated only after  $U(1)_{Z'}$  breaking. The magnitude of the proton decay mediating operators is therefore proportional to the  $U(1)_{Z'}$  breaking scale,  $\Lambda_{Z'}$  which is in turn constrained by the proton lifetime limit. On the other hand, the type of  $U(1)$  that we consider here do not forbid quark, lepton and seesaw mass terms.

By studying the spectrum and symmetries of the string model of ref. [14] Pati showed [9] that  $U(1)$  symmetries with the required properties do indeed exist in the string models. In this paper I examine whether the  $U(1)$  symmetries can remain unbroken down to low, or intermediate, energy scale. This is achieved by examining if there exist supersymmetric flat directions which preserve the specific  $U(1)$  combinations, and hence allow them to remain unbroken down to low, or intermediate energies. In the model of ref. [14] I show that, in fact, such flat directions do not exist. I then study the same question in other models and show that in some examples the required symmetries cannot be preserved by the flat directions, whereas in some cases they can. Imposing the proton lifetime limits I estimate the scale of  $U(1)_{Z'}$  breaking,  $\Lambda_{Z'}$ . I show that in the absence of large R-parity violation  $\Lambda_{Z'}$  is not constrained to be within the reach of forthcoming accelerator experiments, whereas if there exists substantial R-parity violation, the  $Z'$  gauge boson is likely to be seen in forthcoming collider experiments.

## 2 Gauge symmetries in free fermionic models

In this section I discuss the general structure of the realistic free fermionic models, and of the additional  $U(1)$  symmetries that arise in these models. It is important to emphasize that the free fermionic heterotic-string formulation yields a large number of three generation models, which possess an underlying  $Z_2 \times Z_2$  orbifold structure, and differ in their detailed phenomenological characteristics. It is therefore important, as elaborated below, to extract the features of the models that are common to this large class of realistic models.

The free fermionic models are constructed by specifying a set of boundary conditions basis vectors and the one-loop GSO projection coefficients [10]. The basis vectors,  $b_k$ , span a finite additive group  $\Xi = \sum_k n_k b_k$  where  $n_k = 0, \dots, N_{z_k} - 1$ , with  $N_{z_k}$  the smallest positive integer such that  $N_{z_k} b_k = \vec{0} \pmod{2}$ . The physical massless states in the Hilbert space of a given sector  $\alpha \in \Xi$ , are obtained by acting on the vacuum with bosonic and fermionic operators and by applying the generalized GSO projections. The  $U(1)$  charges,  $Q(f)$ , with respect to the unbroken Cartan generators of the four dimensional gauge group, which are in one to one correspondence with the  $U(1)$  currents  $f^* f$  for each complex fermion  $f$ , are given by:

$$Q(f) = \frac{1}{2}\alpha(f) + F(f), \quad (1)$$

where  $\alpha(f)$  is the boundary condition of the world-sheet fermion  $f$  in the sector  $\alpha$ , and  $F_\alpha(f)$  is a fermion number operator counting each mode of  $f$  once (and if  $f$  is complex,  $f^*$  minus once). For periodic fermions,  $\alpha(f) = 1$ , the vacuum is a spinor in order to represent the Clifford algebra of the corresponding zero modes. For each periodic complex fermion  $f$  there are two degenerate vacua  $|+\rangle, |-\rangle$ , annihilated by the zero modes  $f_0$  and  $f_0^*$  and with fermion numbers  $F(f) = 0, -1$ , respectively.

The four dimensional gauge group in the three generation free fermionic models arises as follows. The models can in general be regarded as constructed in two stages. The first stage consists of the NAHE set of boundary conditions basis vectors, which is a set of five boundary condition basis vectors,  $\{\mathbf{1}, S, b_1, b_2, b_3\}$  [15]. The gauge group after imposing the GSO projections induced by the NAHE set basis vectors is  $SO(10) \times SO(6)^3 \times E_8$  with  $N = 1$  supersymmetry. The space-time vector bosons that generate the gauge group arise from the Neveu-Schwarz sector and from the sector  $\mathbf{1} + b_1 + b_2 + b_3$ . The Neveu-Schwarz sector produces the generators of  $SO(10) \times SO(6)^3 \times SO(16)$ . The sector  $\zeta \equiv \mathbf{1} + b_1 + b_2 + b_3$  produces the spinorial **128** of  $SO(16)$  and completes the hidden gauge group to  $E_8$ . At the level of the NAHE set the sectors  $b_1, b_2$  and  $b_3$  produce 48 multiplets, 16 from each, in the 16 representation of  $SO(10)$ . The states from the sectors  $b_j$  are singlets of the hidden  $E_8$  gauge group and transform under the horizontal  $SO(6)_j$  ( $j = 1, 2, 3$ ) symmetries. This structure is common to all the realistic free fermionic models. At this stage we anticipate that the  $SO(10)$  group gives rise to the Standard Model group factors,

whereas the  $SO(6)^3$  groups may produce additional symmetries that can play a role in safeguarding the proton lifetime.

The second stage of the free fermionic basis construction consists of adding to the NAHE set three (or four) additional boundary condition basis vectors. These additional basis vectors reduce the number of generations to three chiral generations, one from each of the sectors  $b_1$ ,  $b_2$  and  $b_3$ , and simultaneously break the four dimensional gauge group. The  $SO(10)$  is broken to one of its subgroups  $SU(5) \times U(1)$ ,  $SO(6) \times SO(4)$ ,  $SU(3) \times SU(2)^2 \times U(1)$  or  $SU(3) \times SU(2) \times U(1)^2$ . Similarly, the hidden  $E_8$  symmetry is broken to one of its subgroups by the basis vectors which extend the NAHE set. This hidden  $E_8$  subgroup may, or may not, contain  $U(1)$  factors which are not enhanced to a non-Abelian symmetry. As the Standard Model states are not charged with respect to these  $U(1)$  symmetries, they cannot play a role in suppressing the proton decay mediating operators, and are therefore not discussed further here. On the other hand, the flavor  $SO(6)^3$  symmetries in the NAHE-based models are always broken to flavor  $U(1)$  symmetries, as the breaking of these symmetries is correlated with the number of chiral generations. Three such  $U(1)_j$  symmetries are always obtained in the NAHE based free fermionic models, from the subgroup of the observable  $E_8$ , which is orthogonal to  $SO(10)$ . These are produced by the world-sheet currents  $\bar{\eta}\bar{\eta}^*$  ( $j = 1, 2, 3$ ), which are part of the Cartan sub-algebra of the observable  $E_8$ . Additional unbroken  $U(1)$  symmetries, denoted typically by  $U(1)_j$  ( $j = 4, 5, \dots$ ), arise by pairing two real fermions from the sets  $\{\bar{y}^{3,\dots,6}\}$ ,  $\{\bar{y}^{1,2}, \bar{\omega}^{5,6}\}$  and  $\{\bar{\omega}^{1,\dots,4}\}$ . The final observable gauge group depends on the number of such pairings.

Subsequent to constructing the basis vectors and extracting the massless spectrum the analysis of the free fermionic models proceeds by calculating the superpotential. The cubic and higher-order terms in the superpotential are obtained by evaluating the correlators

$$A_N \sim \langle V_1^f V_2^f V_3^b \cdots V_N \rangle, \quad (2)$$

where  $V_i^f$  ( $V_i^b$ ) are the fermionic (scalar) components of the vertex operators, using the rules given in [26]. Generically, correlators of the form (2) are of order  $\mathcal{O}(g^{N-2})$ , and hence of progressively higher orders in the weak-coupling limit. Typically, one of the  $U(1)$  factors in the free-fermion models is anomalous, and generates a Fayet-Ilioupoulos term which breaks supersymmetry at the Planck scale. The anomalous  $U(1)$  is broken, and supersymmetry is restored, by a non-trivial VEV for some scalar field that is charged under the anomalous  $U(1)$ . Since this field is in general also charged with respect to the other anomaly-free  $U(1)$  factors, some non-trivial set of other fields must also get non-vanishing VEVs  $\mathcal{V}$ , in order to ensure that the vacuum is supersymmetric. Some of these fields will appear in the nonrenormalizable terms (2), leading to effective operators of lower dimension. Their coefficients contain factors of order  $\mathcal{V}/M \sim 1/10$ . Typically the solution of the D- and F-flatness constraints break most or all of the horizontal  $U(1)$  symmetries. The aim of this paper is to examine whether the  $U(1)$ , proton safeguarding, symmetries can remain unbroken in the supersymmetric vacuum.

### 3 Proton decay and superstring $Z'$ 's

The proton decay mediating terms in a supersymmetric theory are the dimension four operators

$$\eta_1 QUD + \eta_2 UDD \quad (1)$$

and the dimension five operators

$$QQQL \text{ and } UUDE \quad (2)$$

where generation indices are suppressed, and where  $Q$ ,  $L$  are the quark and lepton  $SU(2)_L$  doublets and  $U$ ,  $D$ , are the two quark  $SU(2)_L$  singlets, and  $E$  is the charged lepton  $SU(2)$  singlet.

In the realistic free fermionic models the dimension four operators are forbidden by the gauged  $B - L$  symmetry. However, they are effectively induced after the spontaneous breaking of the  $B - L$  symmetry, from the terms that include the neutral lepton  $SU(2)$  singlet,  $N$ , which is the Standard Model singlet field in the 16 representation of  $SO(10)$ ,

$$QLDN + UDDN . \quad (3)$$

The VEV of  $N$  then induces the effective dimension four operators with effective Yukawa couplings  $\eta \sim \langle N \rangle / M_{\text{string}}$ . The important point is that in the string models, in the absence of the 126 representation of  $SO(10)$ , the  $B - L$  symmetry is necessarily broken at a high scale in order to suppress the left-handed neutrino masses. This breaking is induced either by the VEV of the right handed neutrino  $N$ , or by a combination of fields that effectively carry the  $B - L$  charge of the right handed neutrino. Thus, the dimension four operators are in general induced at some order of nonrenormalizable terms. While it is not impossible that the order will be sufficiently large so as to sufficiently suppress the proton decay, it will clearly be a property of a very specific point in the string moduli space and not a very robust explanation for the proton lifetime.

The problem with proton decay is rather generic in string derived models in which the Standard Model spectrum possess an underlying  $SO(10)$  embedding due to the quartic 16 operator that exist in  $SO(10)$ . Thus, the same problem persists in flipped  $SU(5)$  string models and in the Pati-Salam string models. In fact, in these cases the problem is worse because in these cases the right-handed neutrino is necessarily used to break the GUT  $SU(5)$  or  $SU(4)$  symmetry.

We expect therefore that in superstring models the gauged  $B - L$  symmetry cannot provide adequate protection for the proton lifetime. The basic claim of this paper, therefore, is that, in addition to the Standard Model gauge group, there should exist an additional  $U(1)_{Z'}$  symmetry, which forbids the proton decay mediating operators, and remains unbroken to intermediate or low energies. These operators can therefore arise only from higher order nonrenormalizable terms in the superpotential that contain fields, which are charged under  $U(1)_{Z'}$ . On the other hand, the  $U(1)_{Z'}$  must

be broken above the electroweak scale, as its associated gauge boson has not been observed experimentally. Consequently, the magnitude of the proton decay mediating couplings are proportional to the  $U(1)_{Z'}$  breaking scale. The proton lifetime limits then impose an upper bound on the scale of  $U(1)_{Z'}$  breaking. In addition to the suppression induced by the  $U(1)_{Z'}$  breaking scale, the couplings may also be suppressed because of the order at which they appear in superpotential. That is the couplings may be forbidden by additional  $U(1)$  symmetries that are broken near the Planck scale and induce suppression factors of order  $(1/10)$ , as discussed in section 2. The magnitude of the effective Yukawa couplings is affected by the order of the nonrenormalizable terms that induce the effective couplings. For example, in the string model of ref. [14], we find that the dimension four and five operators can arise from the order sixth terms\*,

$$\begin{aligned}
& (u_3 d_3 + Q_3 L_3) d_2 N_2 \Phi_{45} \bar{\Phi}_2^- \\
& + (u_3 d_3 + Q_3 L_3) d_1 N_1 \Phi_{45} \Phi_1^+ \\
& + u_3 d_2 d_2 N_3 \Phi_{45} \bar{\Phi}_2^- + u_3 d_1 d_1 N_3 \Phi_{45} \Phi_1^+ \\
& + Q_3 L_1 d_3 N_1 \Phi_{45} \Phi_3^+ + Q_3 L_1 d_1 N_3 \Phi_{45} \Phi_3^+ \\
& + Q_3 L_2 d_3 N_2 \Phi_{45} \bar{\Phi}_3^- + Q_3 L_2 d_2 N_3 \Phi_{45} \bar{\Phi}_3^-.
\end{aligned} \tag{4}$$

and

$$Q_3 Q_2 Q_2 L_3 \Phi_{45} \bar{\Phi}_2^- \quad \text{and} \quad Q_3 Q_1 Q_1 L_3 \Phi_{45} \Phi_1^+ \tag{5}$$

respectively, and additional terms are expected to arise at higher orders. Similar terms are found in the other realistic free fermionic models. If we assume a GUT scale VEV for  $N$ , and  $1/10$  suppression factors induced by the other VEVs, we note that the effective dimension four and five operators are not sufficiently suppressed, even if we consider generational mixing.

The question is then whether there exist string symmetries, which are beyond the GUT symmetries and can provide an appealing explanation for the proton lifetime. In a beautifully insightful paper [9] Pati studied this question in the model of ref [14], for the specific choice of the  $U(1)$  combinations that was given in [14], and showed that such symmetries indeed exist in the string models. The question that is studied here is whether the required symmetries can in fact remain unbroken below the string scale, and hence provide the needed suppression. The model of ref. [14] contains six anomalous  $U(1)$  symmetries:  $\text{Tr}U_1 = \text{Tr}U_2 = \text{Tr}U_3 = 24, \text{Tr}U_4 = \text{Tr}U_5 = \text{Tr}U_6 = -12$ . These can be expressed by one anomalous combination which is unique and five non-anomalous ones†:

$$U_A = \frac{1}{\sqrt{15}}(2(U_1 + U_2 + U_3) - (U_4 + U_5 + U_6)) ; \quad \text{Tr}Q_A = \frac{1}{\sqrt{15}}180 . \tag{6}$$

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\*for the notation and charges see ref. [14]

† The normalization of the different  $U(1)$  combinations is fixed by the requirement that the conformal dimension of the massless states still gives  $\bar{h} = 1$  in the new basis.

The choice for the five anomaly-free combinations in ref. [14] is given by

$$U_{12} = \frac{1}{\sqrt{2}}(U_1 - U_2) \quad , \quad U_\psi = \frac{1}{\sqrt{6}}(U_1 + U_2 - 2U_3), \quad (7)$$

$$U_{45} = \frac{1}{\sqrt{2}}(U_4 - U_5) \quad , \quad U_\zeta = \frac{1}{\sqrt{6}}(U_4 + U_5 - 2U_6), \quad (8)$$

$$U_\chi = \frac{1}{\sqrt{15}}(U_1 + U_2 + U_3 + 2U_4 + 2U_5 + 2U_6). \quad (9)$$

The charges of the three generations,  $G_\alpha = E_\alpha + U_\alpha + N_\alpha + D_\alpha + Q_\alpha + L_\alpha$  ( $\alpha = 1, \dots, 3$ ), under the six unrotated  $U(1)^{1,\dots,6}$  are given below

$$(E + U)_{\frac{1}{2},0,0,\frac{1}{2},0,0} + (D + N)_{\frac{1}{2},0,0,-\frac{1}{2},0,0} + (L)_{\frac{1}{2},0,0,\frac{1}{2},0,0} + (Q)_{\frac{1}{2},0,0,-\frac{1}{2},0,0} \quad , \quad (10)$$

$$(E + U)_{0,\frac{1}{2},0,0,\frac{1}{2},0} + (N + D)_{0,\frac{1}{2},0,0,-\frac{1}{2},0} + (L)_{0,\frac{1}{2},0,0,\frac{1}{2},0} + (Q)_{0,\frac{1}{2},0,0,-\frac{1}{2},0} \quad , \quad (11)$$

$$(E + U)_{0,0,\frac{1}{2},0,0,\frac{1}{2}} + (N + D)_{0,0,\frac{1}{2},0,0,-\frac{1}{2}} + (L)_{0,0,\frac{1}{2},0,0,\frac{1}{2}} + (Q)_{0,0,\frac{1}{2},0,0,-\frac{1}{2}} \quad . \quad (12)$$

where<sup>‡</sup>

$$E \equiv [(1, 3/2); (1, 1)]; \quad U \equiv [(\bar{3}, -1/2); (1, -1)]; \quad Q \equiv [(3, 1/2); (2, 0)] \quad (13)$$

$$N \equiv [(1, 3/2); (1, -1)]; \quad D \equiv [(\bar{3}, -1/2); (1, 1)]; \quad L \equiv [(1, -3/2); (2, 0)] \quad (14)$$

of  $SU(3)_C \times U(1)_C \times SU(2)_L \times U(1)_L$ .

$U(1)_\chi$  forbids the terms  $UUDE$  and  $LLDN$  but permits some  $QLDN$ ,  $UDDN$  and  $QQQL$  terms. Therefore, if  $U(1)_\chi$  remains unbroken down to low energies, it does not allow  $R$ -parity violation without inducing rapid proton decay. One must still insure that the dimension four and five operator, which are allowed by  $U(1)_\chi$  are sufficiently suppressed.

$U(1)_\psi$  forbids all the proton decay mediating operators. Thus, provided that  $U(1)_\psi$  remains unbroken down to low energies, the proton decay mediating operators may be sufficiently suppressed. The viability of  $U(1)_\psi$  as a symmetry which sufficiently suppresses the proton decay mediating operators depends on the  $U(1)_\psi$  symmetry breaking scale. The required scale of  $U(1)_\psi$  breaking can be estimated by taking the relevant Yukawa couplings to be a function of the  $U(1)_\psi$  breaking VEVs. On the other hand,  $U(1)_\chi$  and  $U(1)_\psi$  do not forbid the type of superpotential terms,  $QU\bar{h}$ ,  $QD\bar{h}$ ,  $LE\bar{h}$ ,  $LN\bar{h}$  and  $N\bar{N}\phi$ , that generate the fermion masses, but may impose some restriction on the textures of the fermion mass matrices.

Examining Other phenomenological aspects of  $U(1)_\psi$ , we note that  $U(1)_\psi$  is family non-universal. General analysis of the fermion mass matrices suggests that the states from the sectors  $b_1$  and  $b_2$  compose the heavy generation whereas  $b_3$  gives rise to the light generation [25]. This means that the  $U(1)_\psi$  combination produces non-universal charges for the two light families. The existence of a gauge boson with non-universal

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<sup>‡</sup> $U(1)_C = 3/2U(1)_{B-L}; U(1)_L = 2U(1)_{T_{3R}}$



couplings for the two light generations is constrained by Flavor Changing Neutral Currents to be above 30 TeV. This problem, however, may be circumvented if we redefine  $U(1)_\psi$  as  $2U_1 - U_2 - U_3$ . With this redefinition the superpotential terms leading to the dimension four operators are still forbidden. However, assuming that the sector  $b_1$  produces the heavy generation and the sectors  $b_2$  and  $b_3$  produce the two light generations, gives rise to universal  $U(1)_\psi$  charges for the two light generation, which are distinct from the heavy generation  $U(1)_\psi$  charges. Thus, phenomenological constraints on the viability of  $U(1)_\psi$  at energy scales accessible to future experiments depend on detailed analysis of the fermion mass spectrum in the string models.

Next I turn to examine whether the symmetries  $U(1)_\chi$  or  $U(1)_\psi$  in the model of ref. [14] can remain unbroken by the choices of F- and D-flat directions. To examine this question we extract the set of Standard Model singlets that are also neutral under  $U(1)_\chi$  and  $U(1)_\psi$ . The set of fields which are neutral under  $U(1)_\chi$  contains  $\{\Phi_{12}, \bar{\Phi}_{12}, \Phi_{23}, \bar{\Phi}_{23}, \Phi_{13}, \bar{\Phi}_{13}\}$ , and  $T_i, \bar{T}_i$ , which transform as 5 and  $\bar{5}$  of the hidden  $SU(5)$  gauge group. Examining the set of charges of these fields, it is seen that all these fields are either neutral or carry positive charge under the anomalous  $U(1)_A$  symmetry. This means that at least one field which is charged under  $U(1)_\chi$  and carries negative charge under  $U(1)_A$  must acquire a non-vanishing VEV in the cancellation of the anomalous  $U(1)_A$  D-term equation. Consequently, in the model of ref. [14],  $U(1)_\chi$  is necessarily broken by the supersymmetric flat directions, and cannot play a useful role in suppressing the proton decay mediating operators. Similarly, the set of Standard Model singlet fields which are neutral under  $U(1)_\psi$  consist of  $\{\Phi_{12}, \bar{\Phi}_{12}\}$  and  $\{\Phi_{1,2,3}^\pm, \bar{\Phi}_{1,2,3}^\pm\}$ . Again there is no solution to the D-term equations. This results because the  $\{\Phi_{1,2,3}^\pm, \bar{\Phi}_{1,2,3}^\pm\}$  states, which carry  $Q_A = \pm 1$  charges, also carry  $Q_{2'} = \mp 2$  charges, whereas the  $\{\Phi_{12}, \bar{\Phi}_{12}\}$  states are neutral under both. Therefore, there cannot be a simultaneous solution for both  $\langle D_A \rangle = 0$  and  $\langle D_{2'} \rangle = 0$ . Therefore, the two symmetries  $U(1)_\psi$  and  $U(1)_\chi$ , in the model of ref. [14], cannot remain unbroken down to low energies and cannot play a role in safeguarding the proton lifetime. The possible reason for this result is that all the flat directions that have been found in these model utilize the  $SO(10)$  singlet field  $\Phi_{45}$ , which seems to be necessary for D-flatness, and is charged under  $U(1)_\psi$  and  $U(1)_\chi$ .

One may contemplate the possibility in this model [14] that a different choice of the anomaly free  $U(1)$ 's may produce a  $U(1)$  that forbids proton decay and can remain unbroken after implementing the F- and D-flatness constraints. Another choice of the anomaly free combinations is with

$$U_{\psi'} = \frac{1}{\sqrt{21}}(3(U_1 + U_2) - 12U_3 - 4(U_4 + U_5 + U_6)) \quad (15)$$

$$U_{\chi'} = \frac{1}{\sqrt{210}}(2(U_1 + U_2) - U_3 + 2(U_4 + U_5 + U_6)). \quad (16)$$

and the other combinations remain the same. The  $U(1)_{\chi'}$  symmetry now forbids all the proton decay mediating operators. However, in this case the only Standard

Model singlets that are neutral under  $U(1)_{\chi'}$  are  $\{N_{1,2}^c, \Phi_{12}, \bar{\Phi}_{12}\}$ , which are either neutral, or carry positive charge under the anomalous  $U(1)_A$  symmetry. So again a solution for the D-flatness constraints cannot exist with an unbroken  $U(1)_{\chi'}$  and it cannot serve as the proton lifetime safeguarding symmetry.

The above discussion illustrates that despite the existence in the string models of  $U(1)$  symmetries that do forbid the proton decay mediating operators, it is not at all apparent that the needed symmetry can remain unbroken below the string scale. This is in fact a welcomed situation because it is seen that the string framework is highly constrained. To exemplify this further we examine the  $U(1)$  symmetries in the FNY model of ref. [12]. In this model<sup>§</sup>, prior to rotating the anomaly into a single  $U(1)_A$ , six of the FNY model's twelve  $U(1)$  symmetries are anomalous:  $\text{Tr } U_1 = -24$ ,  $\text{Tr } U_2 = -30$ ,  $\text{Tr } U_3 = 18$ ,  $\text{Tr } U_5 = 6$ ,  $\text{Tr } U_6 = 6$  and  $\text{Tr } U_8 = 12$ . Thus, the total anomaly can be rotated into a single  $U(1)_A$  defined by

$$U_A \equiv -4U_1 - 5U_2 + 3U_3 + U_5 + U_6 + 2U_8. \quad (17)$$

The five orthogonal linear combinations,

$$\begin{aligned} U'_1 &= 2U_1 - U_2 + U_3 \quad ; \quad U'_2 = -U_1 + 5U_2 + 7U_3 \quad ; \\ U'_3 &= U_5 - U_6 \quad ; \quad U'_4 = U_5 + U_6 - U_8 \\ U'_5 &= 12U_1 + 15U_2 - 9U_3 + 25U_5 + 50U_8 \quad . \end{aligned} \quad (18)$$

are all traceless. Note that in this case the anomalous  $U(1)$  is not family universal. This arises because of the contribution to the anomaly of the “Wilsonian” sectors beyond the NAHE set. Therefore, it is not a priori apparent that symmetries like the  $U(1)_\chi$  and  $U(1)_\psi$  can exist in this model. Nevertheless, it is seen that in this model, for example,  $U'_1$  forbids all the operators that can induce the dimension four and five proton decay mediating operators. Furthermore, if we assume that the states from the sector  $b_1$  form the heavy generation, while those from  $b_2$  and  $b_3$  give rise to the two light generations, the charges of the two light generations are universal. Therefore, the  $Z'$  gauge boson associated with this symmetry is not strongly constrained by FCNC and could exist at energy scales accessible to future colliders. Similarly, we find that the  $U'_2$  and  $U'_5$  symmetries in this model forbid all the operators that can lead to proton decay, whereas  $U'_3$  and  $U'_4$  and the unrotated  $U_4$  do not. The two symmetries  $U'_2$  and  $U'_5$  are family non-universal and therefore the associated  $Z'$ s are constrained to be heavier than  $\sim 30\text{TeV}$ . A priori, unlike the case of the previous model, it is not apparent that  $U'_2$  or  $U'_5$  cannot survive the D-flatness constraints. However, a general classification of the F- and D-flat directions in the FNY model, did not produce a vacuum in which either of those is preserved [17]. However, the vacua analyzed in ref. [17] included stringent flat directions, which imposes that they are flat to all orders of non-renormalizable terms in the superpotential. Allowing F-flatness breaking at a finite order may yield less restrictive constraints.

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<sup>§</sup>The states and charges of the FNY model are given in ref. [12, 17].

It is instructive to examine the same problem in the model of ref. [16]. This model gives rise to custodial symmetries [24] which forbid the proton decay mediating operators to all orders of nonrenormalizable terms. However, as discussed above, the custodial symmetries, of the type that arise in the model of ref. [16] may be too restrictive and prevent application of the seesaw mechanism. The question that we want to explore is whether this model can allow a symmetry like  $U(1)_\psi$  to remain unbroken by the supersymmetric flat directions. The structure of this model is similar to that of ref. [14]. The model contains three anomalous  $U(1)$  symmetries:  $\text{Tr}U_1 = 24$ ,  $\text{Tr}U_2 = 24$ ,  $\text{Tr}U_3 = 24$ . One combination remains anomalous and is given by:

$$U_A = U_1 + U_2 + U_3, \quad \text{Tr}Q_A = 72. \quad (19)$$

And the two orthogonal combinations can be taken as:

$$U'_1 = U_1 - U_2, \quad U'_2 = U_1 + U_2 - 2U_3. \quad (20)$$

As in the model of ref. [14],  $U'_2$  forbids the terms that can induce the proton decay mediating operators, whereas  $U'_1$  does not. However, it is again found that the model does not admit flat directions that can leave  $U'_2$  unbroken at low energies. The reason being that in this model  $\Phi_{45}$ , which is charged under  $U'_2$  must acquire a non-vanishing VEV in the cancellation of the anomalous  $U(1)_A$  D-term equation.

The above discussion demonstrates that despite the fact that symmetries which forbid the proton decay mediating operators are abundant in the string models, it is not at all apparent that they can remain unbroken down to low energies, and hence fulfill the task of safeguarding the proton lifetime.

Similar results may be expected in the flipped  $SU(5)$  [11] and Pati-Salam type [13] string models. These models share the underlying  $SO(10)$  structure, which is also possessed by the string standard-like models studied above. The dimension four and five, proton decay mediating operators arise from the quartic 16 operator in  $SO(10)$  and are therefore common in all these models. In the flipped  $SU(5)$  the field assignment, in terms of  $SU(5) \times U(1)$  representations, is  $F = (10, 1/2) \in \{Q, D, E\}$ ;  $\bar{f} = (\bar{5}, -3/2) \in \{U, L\}$ ; and  $\ell^c = (1, 5/2) \in \{E\}$ . The dimension four and five, baryon and lepton number violating operators, then arise from  $\{QQQL, QLDN, UDDN\} \rightarrow FFF\bar{f}$  and  $\{UUDE, LLEN\} \rightarrow \bar{f}\bar{f}\ell^c F$ . In the Pati-Salam type string models the field assignment, in terms of the  $SU(4) \times SU(2)_L \times SU(2)_R$  representations is:  $F_L = (4, 2, 1) \in \{Q, L\}$ ; and  $F_R = (\bar{4}, 1, 2) \in \{U, D, E, N\}$ . The proton decay mediating operators then arise from  $\{QQQL\} \rightarrow F_L F_L F_L F_L$ ,  $\{UDDN, UUDE\} \rightarrow F_R F_R F_R F_R$ , and  $\{QLDN, LLEN\} \rightarrow F_L F_L F_R F_R$ . Furthermore, the existence of an anomalous  $U(1)_A$ , which primarily arises from the breaking pattern of  $E_6 \times SO(10) \times U(1)_A$ , is also common to these models. Thus, it may be expected, although not proven, that the symmetries like  $U(1)_\psi$  above, are in general broken near the string scale in this class of models.

To show that indeed the required symmetries can in fact survive down to low energies I turn to the left-right symmetric models of ref. [18]. The unique feature of

these models, in contrast to the standard-like, the flipped  $SU(5)$  and the Pati-Salam type string models, is that the anomalous  $U(1)_A$  does not arise from the symmetry breaking pattern  $E_6 \times SO(10) \times U(1)_A$  [18]. First, I recap the field theory content of these models. The observable sector gauge symmetry is  $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ . Such models are reminiscent of the Pati-Salam type string models, but differ from them by the fact that the  $SU(4)$  gauge group is broken to  $SU(3) \times U(1)_{B-L}$  already at the string level. Similar to the Pati-Salam models [23], the left-right symmetric models possess the  $SO(10)$  embedding. The quarks and leptons are accommodated in the following representations:

$$Q_L^i = (3, 2, 1)_{\frac{1}{6}} = \begin{pmatrix} u \\ d \end{pmatrix}^i \quad (21)$$

$$Q_R^i = (\bar{3}, 1, 2)_{-\frac{1}{6}} = \begin{pmatrix} d^c \\ u^c \end{pmatrix}^i \quad (22)$$

$$L_L^i = (1, 2, 1)_{-\frac{1}{2}} = \begin{pmatrix} \nu \\ e \end{pmatrix}^i \quad (23)$$

$$L_R^i = (1, 1, 2)_{\frac{1}{2}} = \begin{pmatrix} e^c \\ \nu^c \end{pmatrix}^i \quad (24)$$

$$h = (1, 2, 2)_0 = \begin{pmatrix} h_+^u & h_0^d \\ h_0^u & h_-^d \end{pmatrix} \quad (25)$$

where  $h^d$  and  $h^u$  are the two low energy supersymmetric superfields associated with the Minimal Supersymmetric Standard Model. The breaking of  $SU(2)_R$  could be achieved with the VEV of  $h$ . However, this will result with too light  $W_R^\pm$  gauge boson masses. Additional fields that can be used to break  $SU(2)_R$  must therefore be postulated. The simplest set would consist of two fields  $H + \bar{H}$  transforming as  $(1, 1, 2)_{-\frac{1}{2}} + (1, 1, \bar{2})_{\frac{1}{2}}$ . When  $H$  and  $\bar{H}$  acquire VEVs along their neutral components  $SU(2)_R \times U(1)_{B-L}$  is broken to the Standard Model weak-hypercharge,  $U(1)_Y$ . The VEV of the Higgs multiplets  $H + \bar{H}$  breaks the  $B - L$  symmetry spontaneously and, in general, induces the dimension four proton decay mediating operators, whereas the dimension five operators pose a danger irrespective of this VEV. Thus, the need for additional symmetries which suppress these terms is again noted. In terms of the  $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  representations, the baryon and lepton number violating operators arise from  $\{QQQL\} \rightarrow Q_L Q_L Q_L L_L$ ,  $\{UDDN, UUDE\} \rightarrow Q_R Q_R Q_R Q_R$ ,  $\{QLDN\} \rightarrow Q_L Q_R L_L L_R$ , and  $\{LLEN\} \rightarrow L_L L_L L_R L_R$ . We can now examine, in the left-right symmetric string models of ref. [18], whether the dangerous operators are still forbidden by symmetries like  $U(1)_\psi$ . The key feature of the left-right symmetric string models which differs from the previous string models discussed above, is the  $U(1)$  charge assignments of the three generation under  $U(1)_{1,2,3}$ . In the flipped  $SU(5)$ , the Pati-Salam, and the standard-like, string models, the charges of a generation from a sector  $b_j$   $j = 1, 2, 3$ , under the corresponding symmetry  $U(1)_j$  are

either  $+1/2$  or  $-1/2$ , for all the states from that sector. In contrast, in the left-right symmetric string models the corresponding charges, up to a sign are,

$$Q_j(Q_L; L_L) = +1/2 \quad ; \quad Q_j(Q_R; L_R) = -1/2, \quad (26)$$

*i.e.* the charges of the  $SU(2)_L$  doublets have the opposite sign from those of the  $SU(2)_R$  doublets. This is in fact the reason that in the left-right symmetric models it was found that, in contrast to the case of the FSU5, PS and standard-like, string models, the  $U(1)_j$  symmetries are not part of the anomalous  $U(1)$  symmetry [18]. We then note, for example, that the

$$U(1)_\zeta = U_1 + U_2 + U_3$$

combination forbids the dimension five operator  $Q_L Q_L Q_L L_L$  and the operator  $Q_R Q_R Q_R L_R$ , which induces the effective dimension four operator  $UDD\langle N\rangle/M_{\text{string}}$ , while it allows the operator  $Q_L Q_R L_L L_R$ , which induces the dimension four operator  $QLD\langle N\rangle/M_{\text{string}}$ . Similarly, the  $U(1)_\psi = U_1 + U_2 - 2U_3$ , which was examined in the case of the standard-like models above, forbid the  $Q_L Q_L Q_L L_L$  and  $Q_R Q_R Q_R L_R$  terms, while it allows the  $Q_L Q_R L_L L_R$  operator. Thus, in these models  $U(1)_\zeta$ , or  $U(1)_\psi$ , can indeed suppress the proton decay amplitude, while it allows for R-parity violation. On the other hand, because  $U(1)_j$  ( $j = 1, 2, 3$ ) are anomaly free the  $U(1)_\zeta$ , or  $U(1)_\psi$ , combinations can remain unbroken down to low energies. Furthermore, it is noted that the  $U(1)$  combinations which protects the proton longevity are not of a GUT origin, but of an intrinsic string origin. Thus, we have the exciting possibility that, for example, R-parity violation may be accompanied with an additional  $Z'$  gauge boson of intrinsic stringy origin. This demonstrates that the additional stringy  $U(1)$  symmetries, that play the role of safeguarding the proton lifetime, can indeed remain unbroken down to low energies.

## 4 Estimate of the $Z'$ mass

The natural question that arises is at what scale can the  $U(1)$  symmetry, which protects the proton lifetime, be broken, while still providing adequate suppression of the dangerous operators. This question, however, is rather model dependent and depends on the order at which the nonrenormalizable terms, which can induce the proton decay mediating operators, appear, and on possible additional suppression due to generational mixing. Therefore, here I only attempt a rough estimate of the required scale, in the case with and without R-parity violation. The dimension four operators that give rise to rapid proton decay,  $\eta_1 UDD + \eta_2 QLD$ , are induced from the non-renormalizable terms of the form

$$\eta_1(UDDN)\Phi + \eta_2(QLDN)\Phi' \quad (1)$$

where,  $\Phi$  and  $\Phi'$  are combinations of fields that fix the string selection rules. The field  $N$  can be the Standard Model singlet in the 16 representation of  $SO(10)$ , or it can be a product of two fields, which effectively reproduces the  $SO(10)$  charges of  $N$  [8]. I take the VEV of  $N$ , which breaks the  $B-L$  symmetry, to be of the order of the GUT scale, *i.e.*  $\langle N \rangle \sim 10^{16}\text{GeV}$ . This is required because the VEV of  $N$  induces the seesaw mechanism, which suppresses the left-handed neutrino masses. The VEVs of  $\Phi$  and  $\Phi'$  then fixes the magnitude of the effective proton decay mediating operators, with

$$\eta'_1 \sim \frac{\langle N \rangle}{M} \left( \frac{\langle \phi \rangle^n}{M^n} \right) \quad ; \quad \eta'_2 \sim \frac{\langle N \rangle}{M} \left( \frac{\langle \phi' \rangle^n}{M^n} \right). \quad (2)$$

I take  $M$  to be the heterotic string unification scale, which is of order  $10^{18}\text{GeV}$ . Similarly, the dimension five proton decay mediating operator  $QQQL$  can effectively be induced from the nonrenormalizable terms

$$\lambda_1 QQQ L(\Phi'') \quad (3)$$

The VEV of  $\phi''$  then fixes the magnitude of the effective dimension five operator to be

$$\lambda'_1 \sim \lambda_1 \left( \frac{\langle \phi'' \rangle^n}{M^n} \right) \quad (4)$$

The experimental limits impose that the product  $(\eta'_1 \eta'_2) \leq 10^{-24}$  and  $(\lambda'_1/M) \leq 10^{-25}$ . Hence, for  $M \sim M_{\text{string}} \sim 10^{18}\text{GeV}$  we must have  $\lambda'_1 \leq 10^{-7}$ , to guarantee that the proton lifetime is within the experimental bounds. Assuming that the dimension four operators are induced at the quintic order, *i.e.* with one additional field, that breaks the proton protecting  $U(1)_{Z'}$  at intermediate energy scale  $\Lambda_{Z'}$ , we have

$$(\eta'_1 \eta'_2) \sim \left( \frac{\langle N \rangle}{M} \right)^2 \left( \frac{\Lambda_{Z'}}{M} \right)^2 \quad (5)$$

Taking  $\langle N \rangle \sim 10^{16}\text{GeV}$  and  $M \sim 10^{18}\text{GeV}$ , we obtain the estimate  $\Lambda_{Z'} \leq 10^8\text{GeV}$ . similarly, from the dimension five operator we obtain the weaker constraint  $\Lambda_{Z'} \leq 10^{11}\text{GeV}$ . Thus, even in the best case scenario  $\Lambda_{Z'}$  is not constrained to be within the reach of forthcoming collider experiments. On the other hand, if there exist sizable R-parity violation, which necessitates one of the dimension four effective couplings, say  $\eta'_1$ , to be of order  $O(10^{-5} - 1)$ , it imposes that the other effective dimension four operator, say  $\eta'_2$ , is of the order  $\eta'_2 \sim O(10^{-19} - 10^{-24})$ . Taking the smaller value for  $\eta'_1$ , and again taking  $\langle N \rangle \sim 10^{16}\text{GeV}$ , this allows for the  $Z'$  breaking scale to be as low as  $10^{-17}M$ , which is clearly too low. However, other suppression factors can arise from generation mixing, and other VEVs which are of the order of the Fayet-Iliopoulos D-term, and produce suppression of the order  $\langle \phi \rangle/M \sim 1/10$ . These additional suppression factors will result in elevating the  $Z'$  breaking scale by two-four orders of magnitude. All in all, existence of R-parity violating operator may well be accompanied by an additional gauge boson of an intrinsic stringy origin.

The exciting prospect is to correlate between R-parity violation and an additional  $Z'$  gauge boson, whose properties depend on the particular string vacuum.

## 5 Conclusions

The structure of the Standard Model spectrum indicates the realization of grand unification structures in nature. On the other hand the proton longevity severely constrains the possible extensions of the Standard Model and serves as a useful guide in attempts to understand the origin of the Standard Model gauge and matter spectrum. The realistic free fermionic heterotic-string models reproduce the grand unification structures that are suggested by the Standard Model and represent the most realistic string models constructed to date. As such the realistic free fermionic models serve as a useful probe to the fundamental characteristics of the possibly true string vacuum, as well as to various properties that the string vacuum should possess in order to satisfy various phenomenological constraints. In this paper, I proposed that proton stability necessitates the existence of an additional  $U(1)$  symmetry, which remains unbroken down to intermediate or low energies. Furthermore, the required symmetry is not of the type that arises in Grand Unified Theories, but is of intrinsic string origin. The realistic free fermionic models do indeed give rise to  $U(1)$  symmetries, which are external to the GUT symmetries, and forbid the proton decay mediating operators. By studying the supersymmetric flat direction I showed that in some cases the required symmetries cannot remain unbroken in the supersymmetric vacuum, whereas in others they can. Estimate of the  $Z'$  mass reveals that if R-parity violating operators with couplings in the range  $O(10^{-5} - 1)$  exist, then the associated  $Z'$  is likely to be seen in forthcoming collider experiments, whereas if the R-parity violating operators are much suppressed, the  $Z'$  is not constrained to be in the accessible energy range. The phenomenology associated with the additional gauge bosons in the string models will be reported in forthcoming publications.

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